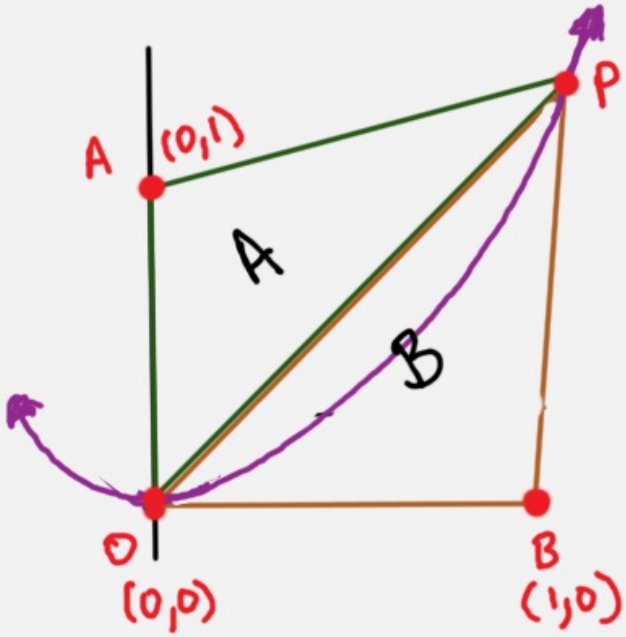
Let $P(x,y)$ be a point on the parabola $y=x^2$ in the 1st quad.
Consider the $\triangle PAO$ and $\triangle PBO$ as seen below



$\triangle PAO$

$AO=1$

$AP=\sqrt{(x-0)^2+(x^2-1)^2}$

$=\sqrt{x^2+x^4-2x^2+1}$

$=\sqrt{x^4-x^2+1}$

$OP=\sqrt{(x-0)^2+(x^2-0)^2}$

$=\sqrt{x^4+x^2}$

$\triangle PBO$

$OB=1$

$BP=\sqrt{(x-1)^2+(x^2-0)^2}$

$=\sqrt{x^2-2x+1+x^4}$

$=\sqrt{x^4+x^2-2x+1}$

$OP=\sqrt{x^4+x^2}$

Perimeter of $\triangle PAO$ with respect to x.

$$P_A(x)=1+\sqrt{x^4-x^2+1}+\sqrt{x^4+x^2}$$

Perimeter of $\triangle PAO$ with respect to x.

$$P_B(x)=1+\sqrt{x^4+x^2-2x+1}+\sqrt{x^4+x^2}$$