

⑥ Find the values of the constants "a" and "b" such that..

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+bx} - \sqrt{3}}{x} = \sqrt{3}$$

$$\frac{\sqrt{a+bx} - \sqrt{3}}{x} \left( \frac{\sqrt{a+bx} + \sqrt{3}}{\sqrt{a+bx} + \sqrt{3}} \right) \rightarrow \frac{a+bx-3}{x(\sqrt{a+bx} + \sqrt{3})}$$

\* using a technique called "conjugate values" helps up simplify to further solving process

still is screwing us up! :o

let  $a=3$

$$\frac{\cancel{3} + bx - \cancel{3}}{x\sqrt{\cancel{3} + bx + \sqrt{3}}} \rightarrow \lim_{x \rightarrow 0} \frac{b}{\sqrt{3+bx} + \sqrt{3}} = \sqrt{3}$$

$$\frac{b}{\sqrt{3} + \sqrt{3}} = \sqrt{3}$$

$$\frac{b}{2\sqrt{3}} = \sqrt{3}$$

$$b = 6$$

So....  
 $a=3$   
 $b=6$

8) Determine all the values of the constant "a" such that the following function is continuous for all Real numbers.

$$f(x) = \begin{cases} \frac{ax}{\tan x} & ; x \geq 0 \\ a^2 - 2 & ; x < 0 \end{cases}$$

$\rightarrow \lim_{x \rightarrow 0^+} \frac{ax}{\tan x} \rightarrow a \lim_{x \rightarrow 0^+} \overset{1}{\cos(x)} \cdot \overset{1}{\left(\frac{x}{\sin x}\right)} = a$   
 $\rightarrow \lim_{x \rightarrow 0^-} a^2 - 2 = a^2 - 2$

Need to be =

$\rightarrow a^2 - 2 = a$

$a^2 - a - 2 = 0$

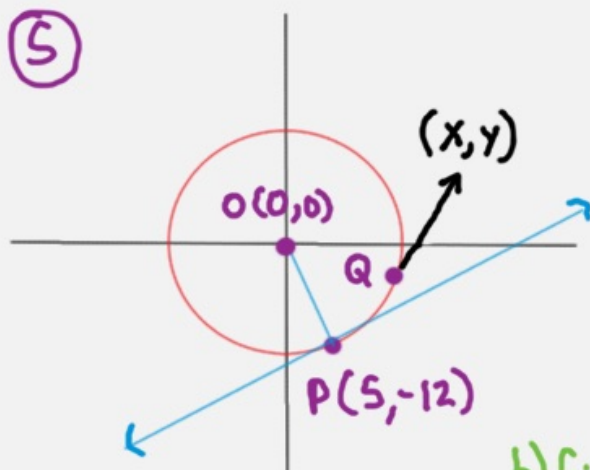
$(a-2)(a+1) = 0$

$a = 2 \quad a = -1$

\*note that special limit can be written

$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$

5



let  $P(5,-12)$  be a point on the circle  $x^2+y^2=169$

a) what is the slope of the tangent line joining O and P?

$$m = -\frac{12}{5}$$

b) find the equation of the tangent line to the circle

$$(y+12) = \frac{5}{12}(x-5)$$

c) Let  $Q(x,y)$  be another point in the fourth quad. Find the slope,  $m_x$  of the line joining P and Q in terms of  $x$ .

$$m = \frac{y - (-12)}{x - 5} = \frac{y+12}{x-5}$$

$$x^2 + y^2 = 169$$

$$y = \pm \sqrt{169 - x^2}$$

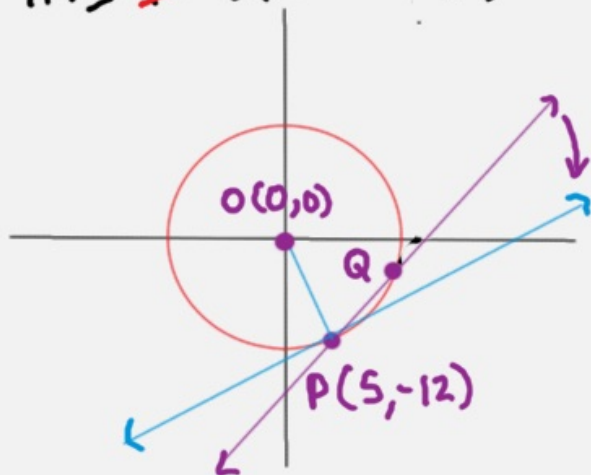
only need bottom half b/c Q is in 4th quad.

$$m = \frac{-\sqrt{169-x^2} + 12}{x-5}$$

d) Calculate  $\lim_{x \rightarrow 5} m_x$  How does this number relate to your answer in part (b)?

$$\lim_{x \rightarrow 5} \frac{12 - \sqrt{169 - x^2}}{x - 5} = \frac{12 + \sqrt{169 - x^2}}{12 + \sqrt{169 - x^2}} = \frac{x^2 - 25 \rightarrow (x-5)(x+5)}{(x-5)(12 + \sqrt{169 - x^2})}$$

$$\lim_{x \rightarrow 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(12 + \sqrt{169 - x^2})} = \frac{5+5}{12 + \sqrt{169 - 25}} = \frac{10}{24} = \frac{5}{12}$$



as "x" approaches 5, the slope of the secant line approaches the slope of tangent line.

